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LETTER TO THE EDITOR

On the consistency of spin-couplings $\frac{3}{2}$

S Deser†

Department of Physics, Brandeis University, Waltham, Massachusettes 02154, USA

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Abstract. Contrary to the implications of a recent paper by Allcock and Hall, spin $\frac{3}{2}$ may be coupled consistently to electromagnetism if gravity is also included; likewise, the gauge variance of the free spin-$\frac{3}{2}$ stress tensor is no hindrance to consistent gravitational coupling.

In a recent paper (Allcock and Hall 1978) on the spin-$\frac{3}{2}$ system, the authors reach negative conclusions concerning the consistency of their gravitational or electromagnetic couplings. We remark here that the difficulties they find are removed within the framework of supergravity.

Consider first electromagnetic coupling. It has been shown (Deser and Zumino 1977) that if (and only if) the full interaction between spin $\frac{3}{2}$, gravitation and electromagnetism is taken into account, then all formal consistency and acausality problems disappear, at least for appropriately related electromagnetic and gravitational coupling constants. This is a remarkable example of the necessity of including all relevant interactions; gravity is needed to cure the flat-space diseases of electromagnetic coupling. Details are given in the cited paper.

The second problem raised is that the free spin-$\frac{3}{2}$ field's stress tensor is gauge variant. This is indeed true, and no more relevant than the similar well known gauge variance of the free massless spin-2 stress tensor is to general relativity (Deser *et al* 1965). For, in the absence of dynamical coupling, the stress tensor is *not* a physical current. Only the integrated values, namely the Poincaré generators, are physical, and these are gauge invariant. On the other hand, when the stress tensor is coupled to gravity, the resulting theory is both fully invariant and consistent. Consider first pure gravity itself. Here the self-coupling leads to a new non-Abelian local gauge invariance, and removes any physical significance from the local 'stress tensor' of the Einstein field (it is just the nonlinear part of the Einstein tensor expanded about flat space), just as for the precisely analogous local isospin current of the Yang–Mills field. There is simply no problem left if one does not demand that arbitrary parts of the gauge field equations be called the 'stress tensor'. Precisely the same thing holds, in a slightly more complicated fashion, in supergravity. The 'stress tensor' of the spin-$\frac{3}{2}$ field is again not invariant, but it is also not physically relevant. The consistency property of supergravity (Deser and Zumino 1976) takes care of the gauge invariance of the coupled Einstein–spin-$\frac{3}{2}$ gauge system. The total energy–momentum of the coupled system is perfectly well defined, and has the proper gauge behaviour.

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We conclude, therefore, that at least in so far as formal properties of spin $\frac{3}{2}$ are concerned, there are no problems whatsoever with electromagnetic and gravitational coupling, and no need to search for exotic formulations of the spin-$\frac{3}{2}$ system. It may well be that real problems arise for elementary spins greater than two, but fortunately these do not (yet) seem to be required by nature either.

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